

## Lecture 1:

In this course all proofs are assumed to be “by empirical data” unless said otherwise.

The universe is in three dimensional space, and each point needs 3 coordinates to represent with a cartesian reference frame. Time can be labelled in an arbitrary reference frame by a real number  $t$ . A point particle is an idealised object that is determined by a position at a given time. We call that  $x(t)$  which is a trajectory a particle follows. It might be an electron or a ball or a planet. The derivative  $x'(t)$  is the velocity  $v$  and we assume this exists. We can write this as  $\dot{x}$ . To differentiate a vector you differentiate each component. The acceleration is the second derivative and we can write this as  $\ddot{x}$  or  $\dot{v}$ .

A free particle is a particle in the universe that is not experiencing any forces, it can be considered to be alone in the depths of space far away from anything else. Suppose the position of this particle is  $x(t)$ .

Definition: An inertial frame is a perspective in which we see a free particle moving at a constant velocity and there is 0 acceleration.

Law of inertia: An inertial frame exists

Gallilean relativity principle: A frame related to an inertial frame by a gallilean transformation is also an inertial frame and all laws of physics are the same in both frames, ie they don't change with time and don't change if I move or turn around, and (perhaps surprisingly) they don't change if I start moving at a constant velocity. A gallilean transformation is  $x' = Rx + k + wt$  where  $R$  is an orthogonal matrix. We call  $w$  a boost and  $k$  a translation. It is easy to check that an inertial frame under such a transformation is an inertial frame.

A boost was historically surprising but for example mass dropped from a boat moving at a constant velocity lands at the same place.

There cannot be any special points in the universe by these principles, neither by space nor by time. Everything is relative. You can be 1 meter away from something but you cannot be 1 meter. However acceleration is not relative. The set of gallilean transformations forms a group.

Under this model all inertial frames have absolute time.

Once there is more than one particle in the universe they will interact. These are described by forces.

Each particle has (inertial) mass  $m$  that we assume is constant. Momentum  $p$  is  $mv$ . Force  $F$  is  $p'$ .  $F$  therefore equals  $ma$ .  $F=ma$  is technically a second order differential equation and technically these newton laws uniquely determine the future. However this has been superceded by other more sophisticated models of physics such as quantum mechanics describing small things and relativity describing big things.

## Lecture 2:

Please use other notes to learn physics I'm genuinely typing this up without understanding anything and if you understand anything I will genuinely be shocked. This is a mess.

We assume forces depend on the position and velocity and that we therefore have a second order differential equation.

Definition: A force is a vector and we say a force is conservative if it can be written as minus the grad of a scalar function  $V$  and we call  $V$  the potential.

Example: The gravitational potential energy of a particle of mass  $m$  at  $x$  due to a particle of mass  $M$  at  $y$  is given by  $V = -\frac{GMm}{|x-y|}$  where  $G \approx 6.67 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$ .

The lecturer did a gradient thing:  $\text{Grad}(x-y)^2$  wrt  $x$  is  $2(x-y)$ .

For reasons I didn't follow we got that  $-\text{Grad}(V)$  is  $-\frac{GMm(x-y)}{|x-y|^3}$  (I think by differentiating  $|x-y|$  by components using pythagoras using  $\frac{d}{dx} \sqrt{x^2+a} = \frac{x}{\sqrt{x^2+a}}$  and doing a chain rule trick). This is the gravitational force.  $F = -\frac{GMm}{r^2} \hat{r}$  with  $\hat{r}$  a unit vector.

Near the surface of the earth if we take  $y=0$  to be the center of the Earth and our particle to be at  $R+z$  where  $R$  is the radius of the Earth.  $-\frac{GMm}{R+z} = -\frac{GMm}{R} \left[ 1 - \frac{z}{R} + \frac{z^2}{R^2} + \dots \right]$  so the potential is about constant near the surface with a difference of about  $\frac{GM}{R^2} z$ . Its gradient is about  $\frac{GM}{R^2}$  which turns out to be about 9.8.

Newtons 2<sup>nd</sup> law says  $m\ddot{x} = mg$  with  $g=(0,0,-g)$ .

We then get that  $x$  is quadratic.

This is ignoring non inertial effects and treating earth as a point particle but 1. We will come back to it and 2. Empirical evidence suggests it is fine.

Conservative forces have a conserved energy.  $E = \frac{1}{2} m \dot{x} \cdot \dot{x} + V(x)$ . The derivative is  $E = \frac{1}{2} m \dot{x} \cdot \ddot{x} + V_{x_i}(x) \cdot \dot{x}$ . This is equal to 0 by Newton (Empirical data, I think).

If we throw an object into space the minimal velocity it must have to never fall down is called the escape velocity. If it is thrown its energy is  $\frac{1}{2} m v^2 - \frac{GMm}{R}$ . To not fall back the object must reach infinity without velocity going to 0. Therefore we need  $\frac{1}{2} m v^2 > \frac{GMm}{R}$  so we can derive it that way. So the escape velocity is  $\sqrt{\frac{2GM}{R}}$ . It is about 10 km / s for Earth. There is no dependence on  $m$ .  $E=T+V$  where  $V$  is the potential energy and  $T$  is the kinetic energy.

Conservative forces have the property that work done by the force as the particle moves along a trajectory  $C$   $W = \text{integral along the trajectory of the force dotted with } dx$  only depends on the end points of the trajectory.

### Lecture 3:

$$\int F \cdot dx = - \int \text{grad}(V) \cdot dx = V(x_1) - V(x_2)$$

A particle can have a charge  $q$ .  $E=Q(e(x)+v \text{ cross } B(x))$

We will restrict to static electromagnetic fields that don't depend on time.

At this point we do some stuff with integrals and electric and magnetic forces that I didn't pay attention to.

**Lecture 4:**

We learned about unit bookkeeping / dimensional analysis and apparently it has to do with analyzing differential equations now I thought it was just unit bookkeeping. Anyway I didn't take other notes because I knew if I had my computer out I wouldn't pay attention and I would get distracted like last time.

**Lecture 5:**

Something about friction and stokes law and terminal velocity FUCK IM TRAUMATIZED FROM A LEVEL PHYSICS EHIRGJALDGHLADFK

**Lecture 6:**

Something about forces that only depend on distance

**Lecture 7:**

Ok this stuff about orbits of planets is actually kind of cool but I stopped paying attention to these lectures long ago

**Lecture 8:**

Something about systems where things are repelling and angles or something. Oh and newtons third law says that the force matrix thingy is antisymmetric.

**Lecture 9:**

Something

**Lecture 10:**

Something about rockets it also looks kinda interesting but I havent been paying attention to these lectures since day 2. Oh and rigid bodies.

**Lecture 11:**

Moment of inertia is  $mr^2$  and we can integrate this

**Lecture 12:**

More rigid bodies stuff then newtons laws in a rotating frame

**Lecture 13:**

Oh shit I forgot to take notes for this one

**Lecture 14:**

Ok so centrifugal forces can happen in a rotational frame something hurricane something fictitious forces

**Lecture 15:**

Light is not relative with gallilean transformations – it goes at the same speed in any inertial reference frame. This was a problem. However, einstein came to the rescue.

We think  $x' = f(x, t)$  and  $t' = g(x, t)$ , then we want it to be that a particle experiencing no forces is at constant speed in any frame but also that speed of light is invariant.

Assume  $x' = ax + bt, t' = cx + dt$ . The invariance postulates constrain a,b,c,d.

Ok I give up on taking notes I'll just try to pay attention